Let $\cos ax/(1+x^{2n})=f(x)$.

$$\int_{-\infty}^{\infty} f(x)dx=\int_0^{\infty} f(x)dx+\int_{-\infty}^{0} f(x)dx=\int_0^{\infty} f(x)dx-\int_{\infty}^{0} f(x)dx=\int_0^{\infty} f(x)dx$$

$$+\int_0^{\infty} f(x)dx=2\int_0^{\infty} f(x)dx.$$

$$\therefore \int_0^{\infty} f(x)dx=\tfrac{1}{2}\int_{-\infty}^{\infty} f(x)dx,=-i\frac{\pi}{2n}\sum_{r=1}^{n}\omega^{2r-1}e^{ai\omega^{2r-1}}.]$$

71. Proposed by J. C. CORBIN, Pine Bluff, Ark.

Form the differential equation of the third order, of which $y=c_1e^{2x}+c_2e^{-3x}+c_3e^x$ is the complete primitive.

I. Solution by EDGAR ODELL LOVETT, Ph. D., Princeton University, Princeton, N. J.

1°. This problem is a familiar one to students of differential equations. The original primitive together with the results of three successive differentiations, may be written

$$\begin{aligned}
&y-e^{2x}c_1-e^{-3x}c_2-e^xc_3=0,\\
&y'-2e^{2x}c_1+3e^{-3x}c_2-e^xc_3=0,\\
&y''-4e^{2x}c_1-9e^{-3x}c_2-e^xc_3=0,\\
&y'''-8e^{2x}c_1+27e^{-3x}c_2-e^xc_3=0\,;
\end{aligned}$$

where $y'\equiv\dfrac{dy}{dx}$, $y''\equiv\dfrac{d^2y}{dx^2}$, $y'''\equiv\dfrac{d^3y}{dx^3}$.

The above is a system of linear and homogeneous equations in the quantities 1, $e^{2x}c_1$, $e^{-3x}c_2$, and e^xc_3, hence the determinant of their coefficients vanishes, that is

$$\begin{vmatrix} y & 1 & 1 & 1\\ y' & 2 & -3 & 1\\ y'' & 4 & 9 & 1\\ y''' & 8 & -27 & 1\end{vmatrix}\equiv\begin{vmatrix} y & 1 & 1 & 1\\ y'-y & 1 & -4 & 0\\ y''-y & 3 & 8 & 0\\ y'''-y & 7 & -28 & 0\end{vmatrix}\equiv 4\begin{vmatrix} y'-y & 1 & 1\\ y''-y & 3 & -2\\ y'''-y & 7 & 7\end{vmatrix}\equiv 0\,;$$

whence

$$\begin{vmatrix} y'-y & 1 & 0\\ y''-y & 3 & -2\\ y'''-y & 7 & 0\end{vmatrix}\equiv 2\begin{vmatrix} y'-y & 1\\ y'''-y & 7\end{vmatrix}=0\,;$$

or finally

$$y'''-7y'+6y=0$$

is the differential equation of the third order whose complete primitive is

$$y-ae^{2x}-be^{-3x}-ce^{x}=0.$$

2°. If the problem be generalized and the complete primitive taken in the form

$$y-ae^{mx}-be^{-(m+n)x}-ce^{nx}=0,$$

the corresponding differential equation of the third order is readily found to be

$$y'''-(m^2+mn+n^2)y'+mn(m+n)y=0.$$

The values $m=2$ and $n=1$ give the original problem.

3°. If the problem be completely generalized and the original primitive taken in the form

$$y-ae^{px}-be^{qx}-ce^{rx}=0,$$

the differential equation is

$$y'''-(p+q+r)y''+(pq+qr+rp)y'-pqry=0.$$

Putting $p+q+r=0$ we have the second case above. If in addition to $p+q+r=0$, $p=2$ and $q=1$, the first particular case appears again.

II. Solution by **WALTER HUGH DRANE, A. M., Professor of Mathematics, Jefferson Military College, Washington, Miss.**

(1) $y=c_1e^{2x}+c_2e^{-3x}+c_3e^{x}$. Differentiate (1).

(2) $\dfrac{dy}{dx}=2c_1e^{2x}-3c_2e^{-3x}+c_3e^{x}$. Subtract (1) from (2).

(3) $\dfrac{dy}{dx}-y=c_1e^{2x}-4c_2e^{-3x}$. Differentiate (3).

(4) $\dfrac{d^2y}{dx^2}-\dfrac{dy}{dx}=2c_1e^{2x}+12c_2e^{-3x}$. Subtract twice (3) from (4).

(5) $\dfrac{d^2y}{dx^2}-3\dfrac{dy}{dx}+2y=20c_2e^{-3x}$. Differentiate (5).

(6) $\dfrac{d^3y}{dx^3}-3\dfrac{d^2y}{dx^2}+2\dfrac{dy}{dx}=-60c_2e^{-3x}$. Add 3 times (5) to (6).

(7) $\dfrac{d^3y}{dx^3}-7\dfrac{dy}{dx}+6y=0$. Q. E. D.

See Johnson's Differential Equations, page 104, example 7.

MECHANICS.

61. Proposed by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy, Ohio University, Athens, Ohio.

A body is suspended from a fixed point by an elastic string, which is stretched to double its natural length when the body is in equilibrium. Find how much the body must be depressed, so that when let go, it may just reach the point of suspension.